National Certificate in Electrical Engineering

Level 2

Problem Solving
Electromechanical Calculations

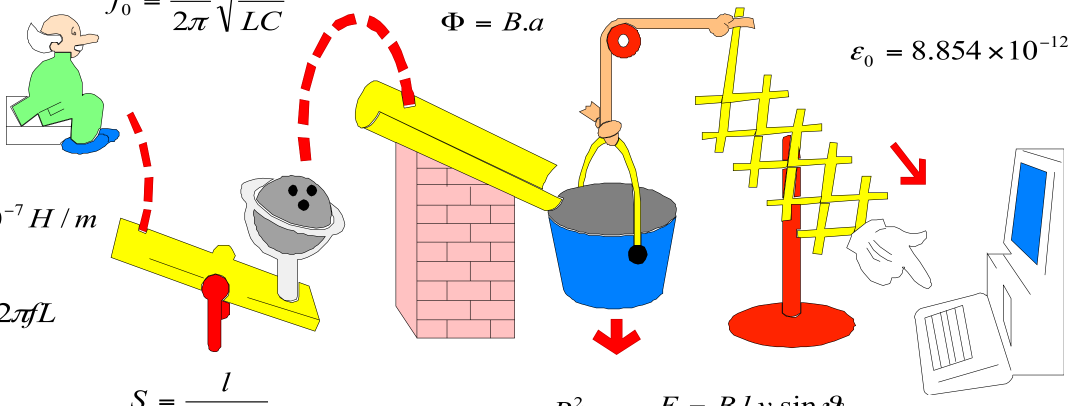
Unit Standard 15847 Version 3

$C = \frac{\varepsilon_0 \varepsilon_r A(n-1)}{d}$ $X_L = 2\pi fL$ $P = \sqrt{3}.V_{Line}.I_{Line}.\cos\vartheta$ $P = V.I.\cos\vartheta$

$\mu = \mu_0 \mu_r$

$X_C = \frac{1}{2\pi fC}$ $f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$ $L = \frac{\mu_0 \mu_r N^2 a}{l}$

$\Phi = B.a$ $\varepsilon_0 = 8.854 \times 10^{-12} F/m$



$SHC_{H^2O} = 4180$

$\mu_0 = 4\pi \times 10^{-7} H/m$

$X_L = 2\pi fL$

$S = \frac{l}{\mu_0 \mu_r a}$ $f = \frac{B^2 a}{2\mu_0}$ $E = B.l.v.\sin\vartheta$

$MMF = S.\phi$ $\rho_{Cu} = 1.72 \times 10^{-8} \Omega m$

$U_m = I.N$ $\frac{R_1}{R_2} = \frac{1+\alpha.t_1}{1+\alpha.t_2}$

You will need:
Calculator – Casio fx-82
Small protractor and Set squares

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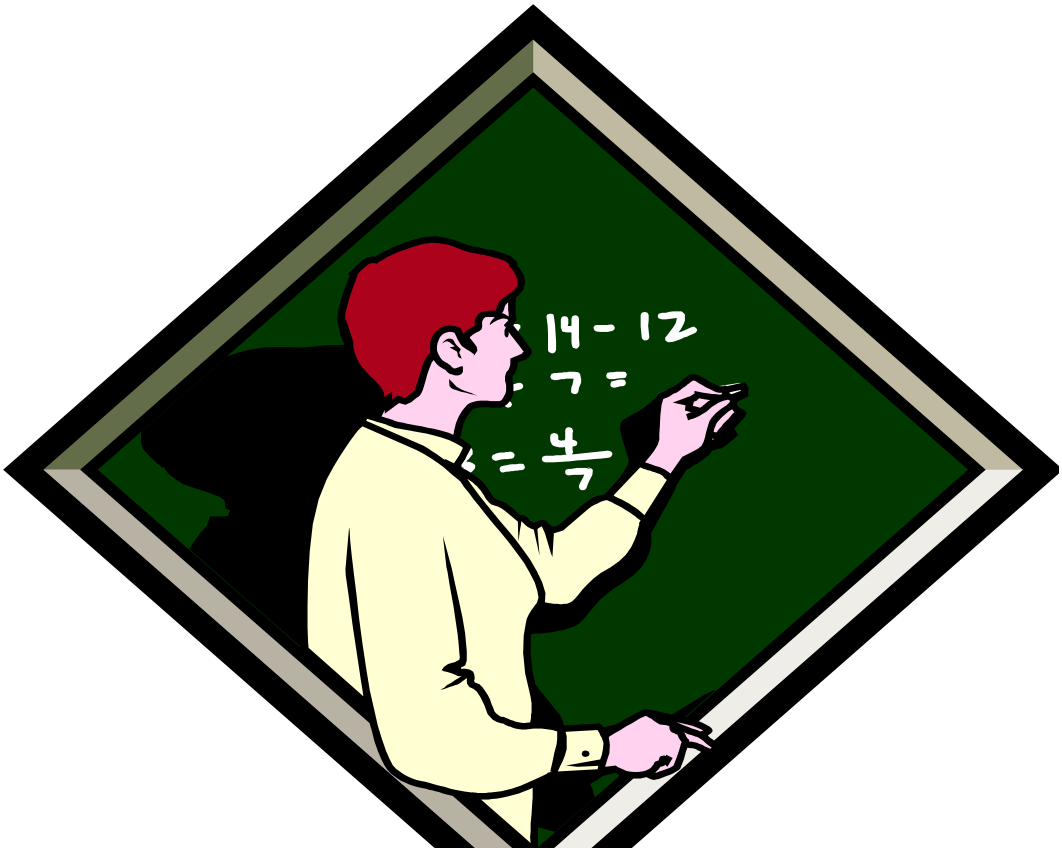
Introduction

This Unit is intended as an introductory maths course for students entering the electrical trades.

For some this course will be simple. For others the course will present difficulties and for these students it may be necessary to supplement the practice exercises with additional problems. Any good final year school text book would provide examples.
It would be appreciated that any errors, omissions or constructive suggestions for improvement be bought to the attention of your tutor to pass onto the writer.

Through your assistance we can improve on this and any handouts for future students. Thanks.



1 - Carry out basic calculations and functions.

A. BASIC ARITHMETIC EQUATIONS

When a number of operations are required, the order in which brackets, exponents, and the basic operations are done is important. Different answers can result if the operations are calculated in different orders.

Example A: The calculation 4 + 2 x 3 involves two operations: addition and multiplication. The answer to the calculation is either 18 or 10, depending on whether the addition is done before or after the multiplication:

The correct answer is 10 because mathematicians have agreed that multiplication is done before addition. Care must be taken when using calculators. Some calculators are programmed to do calculations in the accepted order and others are not.

Throughout the world there is an accepted order for doing calculations involving brackets, exponents, and the four basic operations. The order can be remembered with the help of the mnemonic (***BEDMAS***):

Start Calculation

- B - operations within brackets are done first.
- E - exponents (powers) are done next.
- D & M - division and multiplication are done in order when reading the calculation from left to right.
- A & S - addition and subtraction are done last and in order when reading the calculation from left to right.

Example 1: Solve: 2 + 5 x 2 = 2 + 10 = 12

Division and multiplication are always completed before addition and subtraction unless brackets are used to over rule the BEDMAS rule.

Example 2: Solve: (2 + 5) x 2 = 7 x 2 = 14

Practice Exercise 1

(a) 567.2 +48.9 =

(b) 7.58 +0.93 =

(c) 3.6 x 12 =

(d) 45 x 0.24 =

(e) 25.9 - 13.4 =

(f) 951 - 679.2 =

(g) 265 ÷ 11 =

(h) 509 ÷25 =

Practice Exercise 2

(a) 3 x 6 - 4 x 2 =

(b) 10 ÷ 5 x 3 =

(c) 7 ÷ 7 + 3 x 2 =

(d) 4 x -2 + 3 =

(e) 5 + 3 - 3 x 2 =

(f) 11 x 2 + 11 x 1 =

(g) (5 - 3) x (2 - 1) =

(h) 10 x (3 - 6) + 3 =

(i) 9 x 8 ÷ (16 - 8) =

(j) (5 + 3) x (10 - 2) ÷ (4 - 2) =

(k) (8 x 6 - 4) ÷ (6 - 3) =

B. FRACTIONS – DECIMALS - PERCENTAGES

Fraction to decimal

To change a fraction to a decimal the numerator (top) is divided by the denominator (bottom).

e.g. 1. Write 3/2 as a decimal. 3/2 = 1.5

2. Write 7/8 as a decimal 7/8 = 0.875

Decimals to Fractions

To change a decimal to a fraction the decimal point is removed by writing the numerator as a whole number then dividing by 10, 100, or 1000 as appropriate. Simplify if possible.

e.g. 1. Write 0.3 as a fraction. 0.3 = 3/10

2. Write 0.35 as a fraction. 0.35 = 35/100 = 7/20

3. Write 0.96 as a fraction. 0.96 = 96/100 = 24/25

Percentages

Percent means out of 100. E.g. 20 % means 20 out of 100.

Fractions and Decimals to Percentages.

Multiply the decimal by 100. But for numbers written initially as fractions, first convert to a decimal, and then multiply by 100%.

e.g. 1. Change 0.25 into a percentage. 0.25 = 0.25 x 100% = 25%

2. Change 5/8 into a percentage. 5/8 = 0.625 = 0.625 x 100% = 62.5%

3. Change 2 1/3 into a percentage. 2 1/3 = 2.33 = 2.33 x 100% = 233%

Percentages to Fractions and Decimals.

To change a percentage into a decimal, the percent is changed to a fraction (out of 100) and then the division is carried out.

e.g. 1. Change 25% into a decimal. 25% = 25/100 = 0.25

2. Change 150% into a decimal. 150% = 150/100 = 1.5

3. Change 32% into a decimal. 32% = 32/100 = 0.32

To change a percentage into a fraction, the percent is changed to a fraction (out of 100) and simplified if possible.

e.g. 1. Change 25% into a fraction. 25% = 25/100 = ¼

2. Change 150% into a fraction. 150% = 150/100 = 1 ½

3. Change 32% into a fraction. 32% = 32/100 = 8/25

Practice Exercise 3

3.1 Convert the following to decimals.

(a) 1/3 ______

(b) 1/2 ______

(c) 2/5 ______

(d) 3/4 ______

(e) 7/4 ______

(f) 5/6 ______

(g) 15% ______

(h) 66% ______

(i) 37.5% ______

(j) 125% ______

3.2 Convert the following to fractions.

(a) 0.666

(b) 0.4

(c) 0.75

(d) 0.125

(e) 1.5

(f) 0.048

(g) 15%

(h) 66%

(i) 37.5%

(j) 125%

(k) 192%

(l) 0.5%

3.3 Convert the following to percentages.

(a) 1/3

(b) 1/2

(c) 2/5

(d) 3/4

(e) 7/4

(f) 5/6

(g) 0.666

(h) 0.4

(i) 0.75

(j) 0.125

(k) 1.5

(l) 0.048

c. METRIC MULTIPLES AND SI UNITS

Communication in the electrical industry and indeed any branch of engineering is greatly eased by the use of a standardised set of units with which to measure physical quantities. The Systeme International d'Unites or S.I. has been adopted internationally for this purpose. The fundamental S.I. units are the metre, the kilogram and the second, together with the ampere and the Kelvin. Other units which are needed can be derived from these five basic ones; for example a Newton is the same as a kilogram metre per second^2 (kg m/s^2).

Many quantities, such as velocity and acceleration, are measured in units which do not have a single name, and are written using units in the above list - kg/m^3 for density is an example.

Decimal multiples and submultiples

Where numbers are inconveniently large or small, a standard set of prefixes is used to indicate multiples or submultiples of the basic units. These are shown below with their symbols and decimal equivalents:

Submultiple	Prefix	Symbol
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f

Multiple	Prefix	Symbol
10^{1}	deca	da
10^{2}	hecto	h
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	peta	P

By using the prefixes and their abbreviations many large and small quantities can be expressed in simpler forms:

4 200 000 V = 4.2MV
368 000 W = 368 kW
0.32 N = 320 mN
0.000 047 F = 47 μF
0 .000000 85 s = 850 ns

There are also a few non-SI units still in frequent use which you should be aware of, the most important ones being the litre (1000 ml^3) and the tonne (1000 kg). 1 Litre of water = 1kg mass.

Indices and Base10

We have 10 fingers/digits and a monetary system based on dollars and cents: Decimal currency. $1 equals 100 cents or 10 x 10 cents.

Decimal has a "Base" which is the number 10. We can represent any number using ten unique units, which are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9: Ten units

(We used to have a base 12 for pounds, shillings and pence which was phased out in 1967)

10 x 10 x 10 x 10 x 10 x 10	= 10^6
10 x 10 x 10 x 10 x 10	= 10^5
10 x 10 x 10 x 10	= 10^4
10 x 10 x 10	= 10^3
10 x 10	= 10^2
So it follows that the next is	= 10^1
And that the next is	= 10^0

Hint: You can put this into your fx-82MS calculator as SHIFT 10^x to prove the above statements x being the weight numeric value of Base 10.

The Indices (Index) or Exponential are a shorthand way of representing numbers and are especially useful for large numbers. It is also worth noting that the Indices associated with the base gives "the weight" of any unit within a numeric value.

e.g. 1001
The number does not mean much until you "quantify" the value by adding what these numbers mean. Say dollars or $.

So you have $1001 in your bank account. There are 2 units used: 0 and 1.
The first unit 1 represents $1000 or $1 x 10 x 10 x 10 or $1 x 10^3

The last unit is also a 1 but has a different "weight" associated with it's position.
$1 x 10^0

A number is defined by the Base used and the weight of the units.

4,000,000	= 4 x 10 x 10 x 10 x 10 x 10 x 10	= 4 x 10^6 (Will be entered into the calculator as 4 EXP 6)
3,000	= 3 x 10 x 10 x 10	= 3 x 10^3 (Will be entered into the calculator as 3 EXP 3)

Remember: the **"x 10^3"** part is "**EXP**" in the calculator, not a times or multiply function.

Converting Metric Units

1. Convert 22000 V to kVolts.
Q How many volts is a kV?
A 1000
As we are going from a small quantity of volts to a larger quantity, kVolts, we must shift the decimal place 3 places to the left.
Answer is 22 kVolts.

2. Convert 495 mAmps to Amps.
Q How many mAmps in an Amp?
A 1000.
As we are converting from a small quantity to a larger one shift the decimal place three places to the left.
Answer is 0.495 Amps.

3 Convert 0.22 MΩ to kΩs.
Q How many kΩ in a MΩ.
A 1000.
As we are converting from a large quantity to a smaller one shift the decimal place three places to the right.
Answer is 220 kΩs.

Practice Exercise 4

1. Write 26000 km as m's.
2. 2. Write 0.23 A as mA's.
3. Write 58 kΩs as Ωs.
4. Write 2m as cms.
5 Write 2.7 mm as dm.
6. Write 20 x 10^{-5} F as μF.
7. Write 90.2 MHz as Hz.
8. Write 250 mm^2 as m^2.
9. Write 3.78 tonnes as gm.
10. Write 600 000 V's as kV's

Powers of a quantity (Indices)

Essentially there are three ways for representing numbers.

• As decimal numbers.	55000	0.00034
• Exponential Form	5.5×10^4	3.4×10^{-4}
• Engineering Notation (S.I. units)	55 k	340μ

Scientific or Exponential form.
Numbers are represented as a single decimal number between 1.0 and 9.9 times a multiplication factor, which is described in terms of it's powers of 10 (Exponent). The multiplication factor is positive for numbers greater than 1 and for numbers less than 1 is negative. Note for numbers between 1.0 and 9.9 the multiplication factor is 10^0.

Converting from decimal to exponential form.

1. Convert 9578 to exponential form.
Q How many decimal places to shift the decimal point to 9.578?
Answer: 3 places
Answer is therefore 9.578×10^3

2. Convert 0.000073 to exponential form.
Q How many decimal places to shift the decimal point to 7.3?
Answer: 5 places
Answer is therefore 7.3×10^{-5}

Practice Exercise 5

5.1 Convert to exponential format.

(a) 0.015 s

(b) 16000 m

(c) 6240 Ω

(d) 0.00005 A

(e) 9910000Ω

(f) 6800 Ω

(g) 0.00105 A

(h) 60×10^{-6}V

(i) 53×10^5 W

(j) 0.23 A

5.2 Convert the following to exponential form.

(a) 7.6 kV

(b) 1.9mm

(c) 0.00056A

(d) 56MΩ

(e) 47pF

(f) 298 μA

(g) 0.22 nF

(h) 55 GHz

(i) 0.69THz

(j) 2.6 dm

Converting to Engineering format (SI units)
Move the decimal point in lots of 3's and convert to the nearest SI units.

Converting from decimal to engineering form.

1. Convert 95789 to exponential form.
Q Shift the decimal point 3 places to the left (to 95.789)
Then shift it 3 places more to the left (0.95789) and select the best answer (Usually 1-3 units in front of the decimal point) ***Answer:*** 3 places
Answer is therefore 95.789×10^{3}, as counting 6 places would be too many.

2. Convert 0.000073 to exponential form.
Q Shift the decimal point 3 places to the right (to 0.073)
then shift it 3 places more to the right (to 73)and select the best answer.
Answer: 6 places
Answer is therefore 73×10^{-6}

Note: Sometimes the engineering format and the exponential format give the same answer.
E.g. 9587 is 9.587×10^{3} exponential format and engineering format.

5.3 Convert 5.1 questions to engineering format.

D. USE OF CALCULATORS

Solve problems including square, square - root, cube root, and percentages using your calculator.

***Note**: All calculators are different, so it isn't possible to write a set of instructions to cover every situation. Reference needs to be made to individual calculators and its associated manual.*

With calculators there is one essential rule. Always check your answer by mental arithmetic to ensure it is approximately correct. If not redo.

Square, Square Roots and Cube Roots

When you square a number you multiply the number by itself. e.g.

$4^2 = 4 \times 4$ $\quad$ $2.5^2 = 2.5 \times 2.5$

$= 16$ $\quad$ $= 6.25$

To perform the square function on a calculator enter

$4\ x^2 =\ 16$ $\quad$ $2.5\ x^2 = 6.25$

The square root is the opposite function of the square. e.g.

$\sqrt{16} = 4$ as $4 \times 4 = 16$

$\sqrt{100} = 10$ as $10 \times 10 = 100$

To perform the square root function on a calculator enter

$16\ \sqrt{} = 4$ $\quad$ $100\sqrt{} = 10$

The cube root is the number multiplied by itself 3 times. eg.

$\sqrt[3]{8} = 2$ as $2 \times 2 \times 2 = 8$

$\sqrt[3]{1000} = 10$ as $10 \times 10 \times 10 = 1000$

To perform the cube root function on a calculator enter

$8\ \sqrt[3]{} = 2$ $\quad$ $1000\ \sqrt[3]{} = 10$

Percentages

To express a fraction a percentage

eg Write 2/3 as a percentage: Enter 2 ÷ 3 %

To find a percentage of a number

eg Find 8% of 150: Enter 150 x 8 %

Practice Exercise 6

6.1 Use your calculator to find: the square; square root & cube root of a to e below. (3 decimal places)

	Square	Square root	Cube root
a. 30			
b. 3.6			
c. 0.052			
d. (5.6×10^2)			
e. 2.59			

6.2 Using your calculator solve the following (remember BEDMAS) (3 decimal places)

(a) $2.6 + 1.2^2 =$

(b) $3.6 - \sqrt{10} =$

(c) $5.2^2 \times 3.6^2 - 4 =$

(d) $100 - \sqrt{10} \times 2^2 =$

(e) $\sqrt[3]{(222 - 5.9)}$

(f) $\sqrt[3]{190} – 22.995 \div 4 =$

(g) $7.6^2 \div 2 – 3(\sqrt{16}) =$

(h) Find the area of a square of cloth whose sides have a length of 1.85m

(i) A rectangle measures 1.85m by 47.3m, what is its area?

(j) Find the side of a square equal in area to a rectangle measuring 13m by 10m

(k) Find the length of a square whose area is

(a) $15m^2$,

(b) 130 km^2.

(l) Write the following as percentages.

(a)5/8

(b) 1 and 2/5

(c) 7/16

(m)Find the GST payable on $896.00

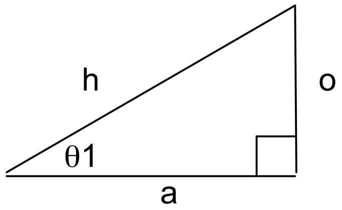
(n) Find 8% of 240.5

(o) Find 0.5% of 796.

E. TRIGONOMETRIC CALCULATIONS.

Pythagoras' theorem

Pythagoras's theorem is only works with right-angled triangles. It is used to find the length of the third side of a triangle given the length of the other two sides.

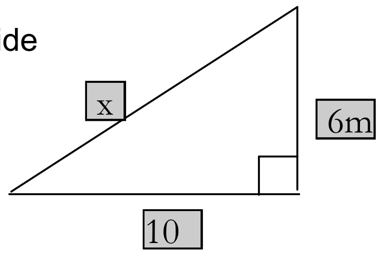


The longest side is called the hypotenuse and given the letter h.
While it is not critical to label the other sides, it is standard convention to label the other 2 sides as being opposite (o) and adjacent (a) in relation to one of the angles. In this case θ1 has o being opposite this angle and a is adjacent to this angle.

$$O^2 + a^2 = h^2$$

Remember: Always the long side will always be h, pick one angle and o will always be opposite it. The left over side will be a, the adjacent side.

Example 2.1. Find the length of the unknown side



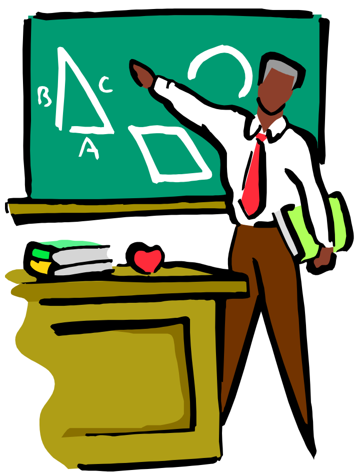
1. Label triangle with o, a, and h.
2. Write formula.
3. Plug in values given on triangle
4. Solve the equation.

Answer:

1. Label triangle.
2. Write formula $\quad h^2 = 6^2 + 10^2$
3. Plug in values $\quad h^2 = 6^2 + 10^2$
4. Solve equation $\quad h^2 = 36 + 100$

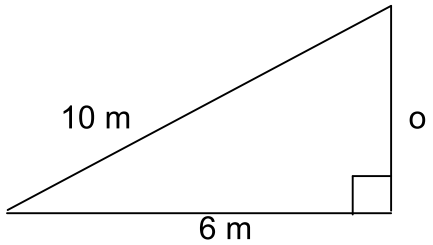
$h = \sqrt{136}$

$h = 11.66m$



Example 2.2

Find the length of the side o.



In this case the formula needs to be rearranged to make o the subject of the formula.

$o^2 + a^2 = h^2$

Can be rewritten as

$o^2 = h^2 - a^2$

$o^2 = h^2 - a^2$
$o^2 = 10^2 - 6^2$
$o^2 = 100 - 36$
$o^2 = 64$
$o = \sqrt{64}$
$o = 8$

Trigonometric Laws or SOH CAH TOA.

These formulas are used to solve right-angled triangles when:

- the length of one side and one angle is known.
- the length of two sides are known.

SOHCAHTOA is just a way to remember the following formulas:

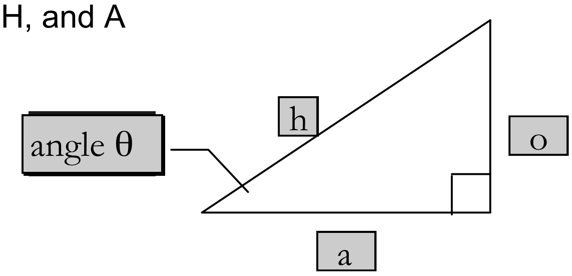
SOH stands for: Sine of angle $\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$

CAH stands for: Cosine of angle $\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$

TOA stands for: Tangent of angle $\theta = \frac{\text{Opposite}}{\text{Adjacent}}$

Remember: The H side is always the longest side (It is always opposite the **right-angle**). The O side is always opposite the angle of interest. The A side is the one left over (called the adjacent side).

Label the triangles below with O, H, and A



Finding lengths and angles using SOHCAHTOA

Finding Lengths:

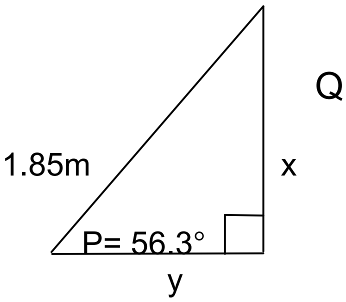
1. Label triangle with O, H and A.
2. Choose the correct formula and write it down first.
3. Put the values in the correct places.
4. Solve the equation and check your answer makes sense.

Finding angles

1. Label triangle with O, H and A. O is opposite the angle of interest.
2. Choose the correct formula and write it.
3. Put the values in the correct places.
4. Solve the equation and check your answer makes sense.

Examples

2.3 Solve for the unknown sides and angles.



$\text{Sin } P = \frac{\text{opp}}{\text{hyp}}$

$\text{Sin } 56.3° = x/1.85$

$x = 1.85 \times \text{Sin } 56.3°$

$= 1.54\text{m}$

To find the remaining angle Q, use the fact that the sum of internal angles in a triangle all add up to 180°, therefore angle Q = 180° – 56.3° – 90° = 33.7°.

To solve for the other side y use either Pythagoras's theorem, alternatively

$\text{Cos } P = \frac{\text{adj}}{\text{hyp}}$

$\text{Cos } 56.3° = \frac{y}{1.85}$

$y = 1.85 \times \text{Cos } 56.3°$

$= 1.03 \text{ m}$

2.4 Solve for the unknown side H and angles (B and C).

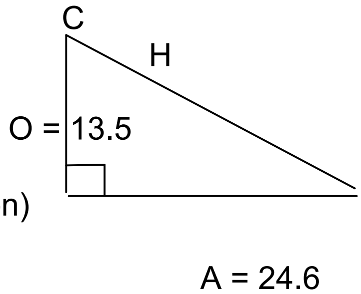
Tan B $= \frac{opp}{adj}$

$= \frac{13.5}{24.6}$

$= 0.5487$

B $= \text{Tan}^{-1}\ 28.8°$ (inverse Tan function)

B



As in example angle C = 180° -90° - 28.8° = 61.2°

To solve for the hypotenuse use either Pythagoras or alternatively,

Cos B $= \frac{24.6}{H}$

H $= \frac{24.6}{\text{Cos } 28.8°}$

$= \frac{24.6}{0.8763}$

$= 28.07$

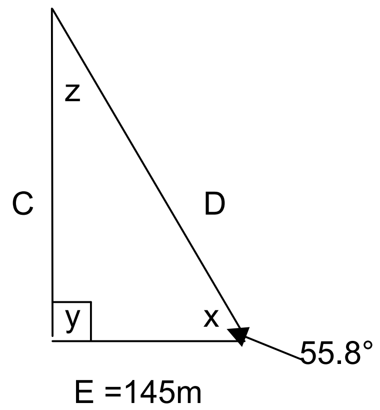
2.5 Solve for the unknown sides and triangle

Cos x $= \frac{adj}{hyp}$

Cos 55.8° $= \frac{145}{D}$

D $= \frac{145}{\text{Cos } 55.8°}$

= 258m



Angle z = 180° -90° - 55.8°

= 34.2°

To find side C

Tan z $= \frac{opp}{Adj}$

Tan 34.2° $= \frac{145}{C}$

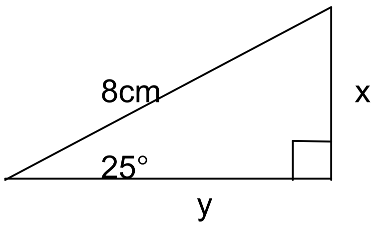
C = 145 /Tan 34.2°

= 213.36 m

Practice Exercise 7

1. Find the length of the unknown sides.

(a)



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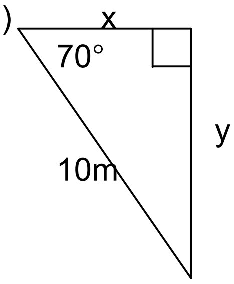
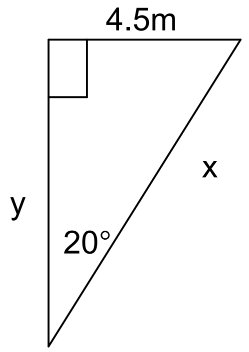
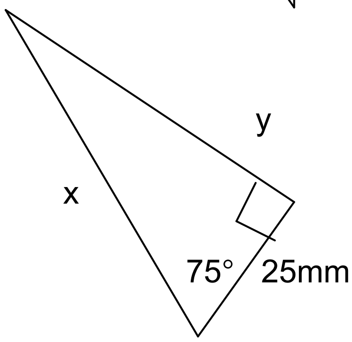


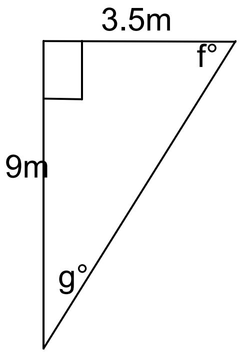
(c)



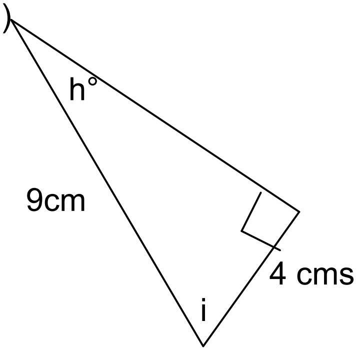


7.2 Find the unknown angles

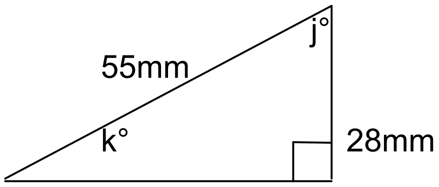
(a)



(b)



(c)



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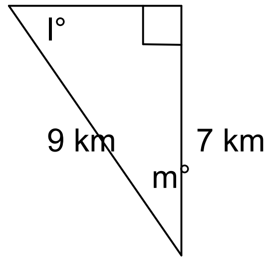


7.3 A stage light is positioned on a beam 20 m above the front of a stage. The light makes an angle of 20° to the horizontal. Determine the length of the light beam.

7.4 A power cable is to run on a diagonal across a 50metre by 20 metre stadium. Determine the length of cable required.

F. TRANSPOSE FORMULA

3.1 The laws for transposing formula are stated.
3.2 Given formula are stated so as to solve for an unknown quantity.
Range: Ohms' Law, formulae for power, force and energy.

The material in this unit is provided to give an overview of the laws associated with this unit. It isn't intended to be an introduction to electrical engineering.
For more detail see any good basic Electrical Engineering text.

Some Basic Electrical Engineering Concepts.

To work in the electrical industry you will need to be able to carry out many and varied electrical calculations. The common formulae are based around the following laws.

OHMS LAW

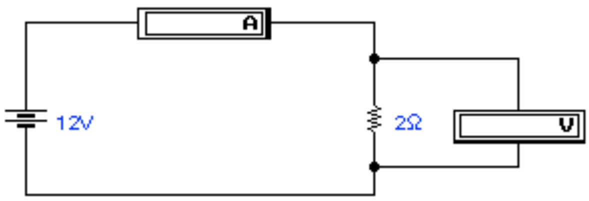
In an electrical circuit the current flowing through the circuit is proportional to the voltage across the circuit, providing the temperature is kept constant, and the circuit total resistance.

or if we double the voltage across a circuit or component the current flowing through the circuit or component doubles. The opposition to current flow is resistance and

Resistance = Volts ÷ Current

$$R = \frac{V}{I}$$

Units for Resistance is ohms symbol Ω.
Current is Amps symbol A.
Voltage is Volts symbol V.



in the above example the resistance of the circuit I is the voltage ÷ by the resistance.

$$I = \frac{V}{R}$$
$$= 12/2$$
$$= 6 \text{ amps}$$

Power in Electric Circuits

The power (symbol P, unit Watts) consumed by an electrical appliance or an electrical circuit is:

$$\text{Power} = \text{Volts} \times \text{Amps}$$
$$P = V \times I$$

in the above example the power consumed by the circuit

$$P = 12 \times 6$$
$$= 72 \text{ Watts}$$

Energy consumed in Electric Circuits

The energy (symbol E unit joules) consumed in electrical systems and indeed mechanical systems is the product of power and time or power is the rate of using energy. The unit t is in Seconds (S)

$$E = P \times t$$

Example: Determine the energy used by a 200 Watt bulb running continuously for 2 hours.

$$E = P \times t$$
$$= 200 \times 2 \times 60 \times 60$$
$$= 1.44\text{M joules}$$

The joule isn't a very practical unit for electrical energy and the kilowatt hour is the preferred unit. Reworking the above example in kWatt.hours. (Convert 200W to kWatts first)

$$E = P \times t$$
$$= 0.2 \times 2$$
$$= 0.4 \text{ kWatt.hours.}$$

Force

In mechanical systems the force (symbol F: unit Newton) on an object is the product of the mass of the object times its acceleration. If acceleration is due to gravity this is 9.81m/s^2

$$F = m \times a$$

Example: Determine the weight of a 2 kg mass.

$$F = m \times a$$
$$= 2 \times 9.8$$
$$= 19.62 \text{ Newtons}$$

Common Formulae in Use

$V = IR$

$P = VI$

$P = I^2R$ (Transpose V=IR into the formula P=VI)

$P = \frac{V^2}{R}$ (Transpose I=V/R into the formula P=VI)

$E = Pt$

$F = ma$

Transposition of Formulae:

Consider the formula $y = 2a + b$

to make b the subject we must shift the 2a to the other side.

As the 2a is positive, to get rid of it from the right hand side of the formula, subtract it from the right hand side. As we subtract it from the right hand side we must also subtract it from the left hand side.

$$y - 2a = 2a - 2a + b$$
$$y - 2a = b$$

Using the same formula and making a the subject.

$$y = 2a + b$$

subtracting b from both sides $y - b = 2a$

dividing both sides by 2 $\frac{y-b}{2} = a$

Rules of Transposition

1. When shifting from one side of an equation to the other, use the opposite function. eg Opposite of multiplication is division, subtraction is addition, squaring is square rooting.

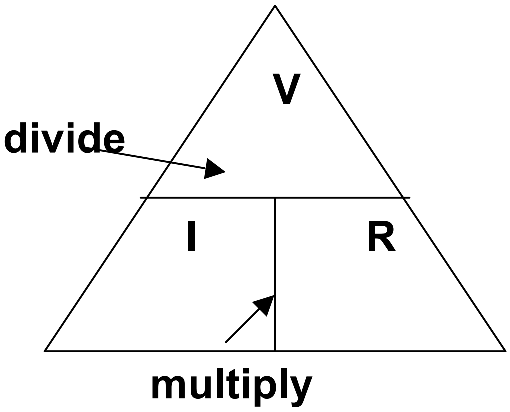
2. To keep the original meaning (balance) of the formula carry out the same mathematical function to both sides.

(Note: Similar to BEDMAS rules but in reverse)

Another Method of Transposition

In some cases we may be asked to solve the equation for a quantity that isn't the subject of the formula. eg If V = IR solve for R. or if P = VI, solve for I

The formulae can be placed in a triangle as follows.



By covering the unknown subject the correct formula can now be read directly from the triangle.
Eg. To find R. Cover R and read it as V over (divided by)I. To find I, cover I and read as V over (divided by) R

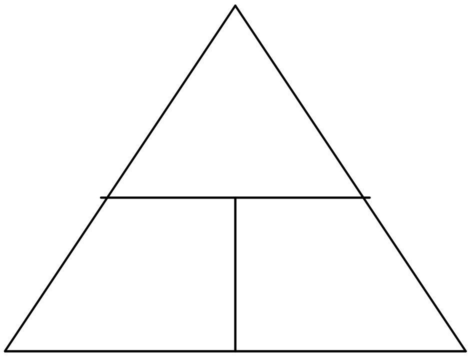
$R = \frac{V}{I}$ $\qquad\qquad$ $I = \frac{V}{R}$

When placing formulae into the triangle ensure that you maintain the relationship contained in the original formulae.

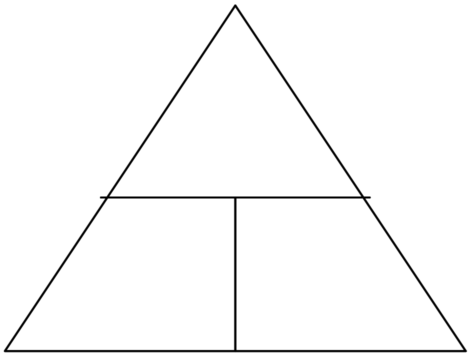
If the formulae is V = IR the IR goes on the bottom line. The V in the remaining slot.
If the formulae is R = V/I the V goes in the top slot and the I underneath. R in the remaining slot.

Practice Exercise 8

1. Place the formula P = VI into the triangle and rearrange the formula in terms of I and V.



2. Repeat for $P = \frac{V^2}{R}$ and rearrange for V^2 and R.



3. Rearrange E = Pt to make both P and then t the subject of the formula. Solve for P if E the energy = 100 joules and t the time = 50 seconds.

4. Rearrange F = ma to make a the subject of the formula. Therefore find the acceleration of a 5 kg object if a force of 10 Newton's is applied to it.

5. Rearrange P = V.I to make I the subject. Determine the current drawn from the 230 V mains by a 200-Watt spotlight.

6. Rearrange E = Pt to make t the subject. Determine the time a 2kW motor is running if it uses 9.5 kJ of energy.

7. Find the voltage across a 5Ω load if it draws a current of 5 Amps.

8. An heater is rated at 500 W. What current will it draw from the 230 V mains.

9. Rearrange E = P x t to make P the subject. Determine the power rating of a pump which uses 2.5 kwatt-hours of energy when operated for 30 mins.

10. Rearrange F = ma to make m the subject. Find the mass of an object which accelerates at 2m/sec^2 when a force of 8.5 Newtons is applied to it.

Dealing with square terms.

When working with power formulae we have to deal with square terms eg V^2 and I^2.

If $P = V^2/R$

$V^2 = PxR$

To solve for V we must take the square root of both sides of the equation. *The opposite of squaring a term is the square root.*

Then $V = \sqrt{PR}$

Similarly if;

$P = I^2R$

$I^2 = P/R$

?

Practice Exercise 9

1. For the formula $P = I^2R$ rearrange to make I the subject and find the current when the power is 2.5 kW and R = 50 Ω's.

2. When $y = \frac{z}{x^2}$ rearrange to make x the subject and solve for x when y = 2.6 and z = 5.2.

3. For the formula $P = V^2/R$ rearrange to make R the subject and use the formula to determine the resistance of a 60 Watt 230 V lamp.

4. Using the formula $P = I^2R$ rearrange to make R the subject and determine the resistance of a lamp filament rated at 460 W which draws a current of 2 Amps.

5. Determine the Voltage across a 1 kW appliance with a resistance of 10Ω's.

6. Find the current drawn by the appliance in Q 5 above.

ANSWER TO PRACTICE EXERCISES

One

(a) 616.1 (b) 8.51 (c) 43.2 (d) 10.8 (e) 12.5 (f) 271.8
(g) 24.09 (h)20.36

Two

(a) 10 (b) 6 (c) 7 (d) -5 (e) 2 (f) 33
(g) 2 (h) -27 (i) 9 (j) 32 (k) 14 2/3 or 14.6666

Three

3.1
(a) 0.33 (b) 0.5 (c) 0.4 (d) 0.75 (e) 1.75 (f) 0.833
(g) 0.15 (h) 0.66 (i) 0.375 (j) 1.25

3.2
(a) 333/500 (2/3) (b) 2/5 (c) 3/4 (d) 1/8 (e) 3/2 (f) 6/125
(g) 3/20 (h) 33/50 (2/3) (i) 3/8 (j) 5/4 (k) 48/25 (l) 1/200

3.3
(a) 33.33% (b) 50% (c) 40% (d) 75% (e) 175% (f) 83.33%
(g) 66.66% (h) 40% (i) 75% (j) 12.5% (k) 150% (l) 4.8%

Four

(1) 26,000,000 m's (2) 230 mA (3) 58000 Ω (4) 200 cms
(5) 0.027 dm (6) 200 μF (7) 90,200,000Hz (8) 250 x $10^{-6}m^2$
(9) 3,780,000 gms (10) 600 kV

Five

5.1
(a) 1.5 x 10^{-2}S (b) 1.6 x 10^{4}m (c) 6.24 x 10^{3}Ω (d) 5 x 10^{-5}A (e) 9.91 x 10^{6}Ω
(f) 6.8 x 10^{3}Ω (g) 1.05 x 10^{-3}A (h) 6 x10^{-5}V (i) 5.3 x10^{6}W (j) 2.3x 10^{-1}A

5.2
(a) 7.6 x 10^{3} V (b) 1.9 x 10^{-3}m (c) 5.6 x 10^{-4} A (d) 5.6 x 10^{7}Ω (e) 4.7 x 10^{-11} F
(f) 2.98 x 10^{-4}A (g) 2.2 x 10^{-10} F (h) 5.5.x 10^{10} Hz (i) 6.9 x 10^{11} Hz
(j) 2.6 x 10^{-1} m

5.3
(a) 15mS (b) 16km (c) 6.24kΩ (d) 50μA (e) 9.91MΩ
(f) 6.8kΩ (g) 1.05mA (h) 60μV (i) 5.3MW (j) 230mA

Six

6.1

	Square	Square root	Cube root
a. 30	900	5.477	3.107
b. 3.6	12.96	1.897	1.533
c. 0.052	$2.7x10^{-3}$	0.228	0.373
d. (5.6×10^2)	313600	23.664	8.243
e. 2.59	6.708	1.609	1.373

6.2 (a) 4.04 (b) 0.4377 (c) 346.438 (d) 87.35 (e) 6.00
(f) 0.00015 (g)16.88 (h) 3.42 m^2 (i) 87.5 m^2 (j) 11.40 m
(k)(a) 3.87m (k)(b) 11.4 km (l)(a) 62.5% (l) (b) 140%
(m) $134.40 (n) 19.24 (o) 3.98

Seven

7.1

(a) x = 3.38cm y = 7.25 cm (b) x = 3.42my = 9.397m (c) x = 13.157m y = 12.36m
(d) x = 96.59mm y = 93.3mm

7.2

(a) f = 68.75° g = 21.25° (b) h = 26.39° I = 63.61° (c)j = 59.4° k = 30.6°
(d) l = 51.06° m = 38.94°

7.3 58.47 m 7.4 53.85 m

Eight

1. I = P/V V = P/I
2. V^2 = PR R = V^2/P
3. P = E/t t = E/P 2 Watts
4. a = F/m a = 2 m/sec^2
5. I = P/V I = 0.869A
6. t = E/P t = 4.75S
7. V = 25 V
8. I = 2.174A
9. P = E/t P = 5 kW
10. m = F/a m = 4.25 kg

Nine

1. $I = \sqrt{\frac{P}{R}}$ 7.07 Amps
2. $X = \sqrt{\frac{Z}{Y}}$ 1.414
3. R = V^2/P 881.67 Ω
4. R = P/I^2 115 Ω
5. V = 100 V
6. I = 10 Amps

G. S.I. UNITS

An International System of Units is used so that people from different countries will all be able to use the same measurements.

There are seven "base" units, and all other units are derived from these:

QUANTITY	UNIT	SYMBOL
length	Metre	m
mass	Kilogram	kg
time	Second	s
electric current	Ampere	A
temperature	Kelvin	K
luminous intensity	Candela	cd
amount of substance	Mole	mol

Table 5. SI prefixes

Factor	Prefix	Symbol	Factor	Prefix	Symbol
$10^{24} = (10^3)^8$	yotta	Y	10^{-1}	deci	d
$10^{21} = (10^3)^7$	zetta	Z	10^{-2}	centi	c
$10^{18} = (10^3)^6$	exa	E	$10^{-3} = (10^3)^{-1}$	milli	m
$10^{15} = (10^3)^5$	peta	P	$10^{-6} = (10^3)^{-2}$	micro	μ
$10^{12} = (10^3)^4$	tera	T	$10^{-9} = (10^3)^{-3}$	nano	n
$10^9 = (10^3)^3$	giga	G	$10^{-12} = (10^3)^{-4}$	pico	p
$10^6 = (10^3)^2$	mega	M	$10^{-15} = (10^3)^{-5}$	femto	f
$10^3 = (10^3)^1$	kilo	k	$10^{-18} = (10^3)^{-6}$	atto	a
10^2	hecto	h	$10^{-21} = (10^3)^{-7}$	zepto	z
10^1	deka	da	$10^{-24} = (10^3)^{-8}$	yocto	y

NOTE: A commonly used derivative of the kilogram is the gram – 1/1000kg = 1 gram.

A change in temperature of 1K is equivalent to a change of 1°C (Celsius).

H. SPEED AND VELOCITY

- Speed and velocity are more or less the same thing the only difference being that velocity is in a given direction.

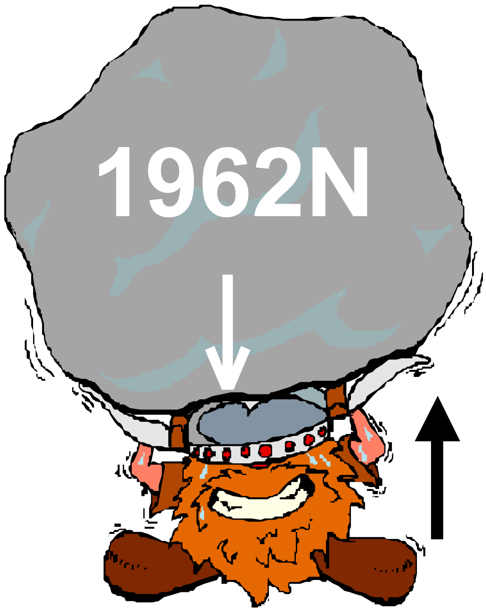
It is quite likely that you measure speed in kilometres per hour (km/h) - this is all very well for giving you an idea of how fast you should approach a speed camera, but not very practical when it comes to solving mechanical problems.

1. The unit for velocity is the quotient (result from division) of two of the SI Units on the previous page - what is the unit for velocity?

Velocity is in a given direction and is measured in:

The symbol of these units is:

I. FORCE



Force is measured in Newtons and, like velocity, is in a given direction. The acceleration due to gravity is $9.81 m/s^2$

The force applied by gravity to the mass of 1kg is 9.81N in the direction of the centre of the earth.

One would hope that the little guy in the picture is exerting a force of 1962N on the rock in the opposite direction to that applied by gravity otherwise he will surely be crushed.

2. What is the mass of the rock (i.e., how much does it "weigh"*)?

kg

Forces can be added "vectorally" (As vectors) - 1.962kN↓ + 1.962kN↑ = 0 N

3. Complete the missing words/symbols in the appropriate spaces below:

Force is in a given direction and is measured in:

The symbol of these units is:

*** Weight is actually measured in Newtons - only mass is measured in kilograms.**

J. ENERGY AND WORK

1) To lift an object from the ground to a certain height you must apply a certain amount of Force to do a corresponding amount of Work. Work is measured in Joules (J).

2) To do that Work you would have to apply a certain amount of Energy equal to the amount of work to be done. Energy is also measured in Joules (J).

3) Once that work was done, and the object was at that certain height, it would now possess a certain amount of **Potential** Energy of its own.

4) Now, if that object fell from that certain height, onto your steel capped safety boot, it would do a certain amount of Work on your toes - unless the rating of the safety boot was sufficient to withstand the amount of Kinetic Energy applied by the falling object.

1) If you were to lift an object weighing 11kg, the Force that you would have to apply would be:

2) To lift this mass 1.8m, you would have to apply this amount of Force over the full 1.8m and that would equate to 194.2 joules of Work.

$$F = m \times g$$
$$= 11kg \times 9.81m/s^2$$
$$= 107.9N$$

3) Now that this 11kg mass is at a height of 1.8m, it has a Potential Energy of 194.2J.

$$W = m \times g \times h$$
$$= 11kg \times 9.81m/s^2 \times 1.8m$$
$$= 194.2J$$

4) If this 11kg mass fell from that height, the acceleration due to gravity would mean that the mass would apply 194.2J of Kinetic Energy to your toes.

Safety shoes - typical impact rating - 200J, so your toes will not be squashed.

Work is done when Energy is converted from one form to another:

- the mass had no Energy when at rest on the ground;
- the mass required Energy from the person to lift it so Work was being done by the person;
- the mass had Potential Energy when resting at a height of 1.8m but no Work was being done; and
- Kinetic Energy (energy due to motion) applied a force at the toecap and Work was done - although not enough to damage the person's toes.

NOTE:

Work done in moving an object horizontally is a result of the force that must be applied to overcome friction and possibly wind etc.

4. How much Work is done lifting an All Black with a mass of 95kg to a height of 1.3m? How much Force must be applied to hold him there?

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5. How much Energy would a marble weighing 10g have just before impact with the ground if dropped from the top of the New Plymouth Power Station chimney (198m)? Is this Potential or Kinetic Energy?

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On the previous page we mentioned that Work is done when Energy is converted from one form to another.

It stands to reason then, that Work is also done when some form of Potential Energy is converted to Heat Energy...

To heat a substance to a certain temperature the Work done can be calculated by the formula:

Work Done **= mass × Specific Heat Capacity × temperature change (∆t)**

(Energy that must have been applied)

NB: The specific heat capacity (SHC) of a substance is related the amount of Energy required to raise the temperature of 1kg of that substance by 1°C.
The SHC for water is 4180 (although there is some variation with temp.)

Answer the following questions:

6. How much Energy is required to raise the temperature of 2 litres of water from 10°C to 100°C (i.e., to boil the jug)? (1 litre of water has a mass of 1kg)

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7. How much Work would have been done if the jug used to heat the water in the question above was 95% efficient (i.e., η = 0.95)?

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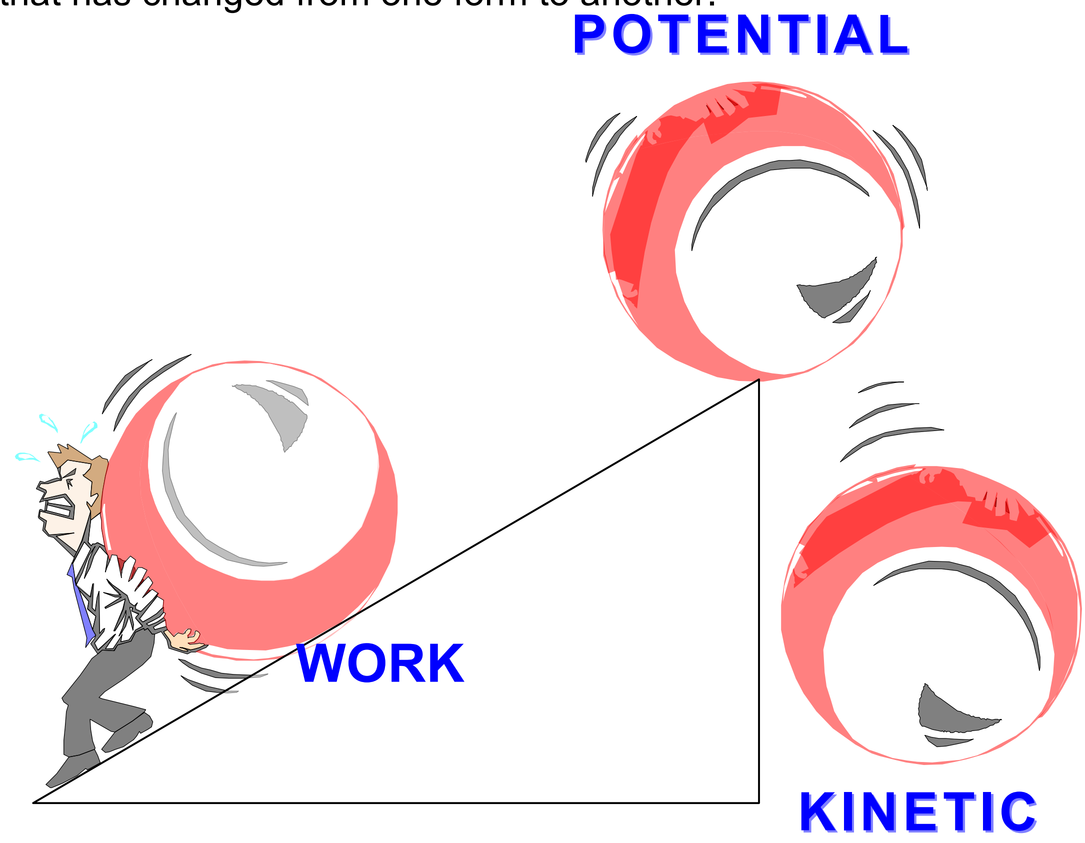
K. POWER

We have so far concluded that a certain amount of Work is done when converting Energy:

A. A body has Potential Energy relative to a lower surface. If the body was released it would lose its Potential Energy and gain an equal amount of Kinetic Energy. The Work Done is the amount of Energy that has changed from one form to another.



B. **Electrical Energy** to Heat Energy (boiling a jug).

Although these are not the only places where Work is done (i.e., Potential Energy is converted to Kinetic Energy when something falls from height, and, Kinetic Energy of one body can be converted to Kinetic Energy of another body when a moving car hits a stationary car setting the second car in motion) - we will consider only the Work and Energy requirements of **A.** and **B.**.

The relationship between Power and Energy…

$$1 \text{ Joule (J)} = 1 \text{ watt (W)} \times 1 \text{ second (s)}$$

Power is measured in watts.

Power is the rate at which Work is done.

Answer the following questions:

8. ***How long would it take a ¼Hp (186.4W) motor to lift an 11kg mass to 1.8m?***

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9. ***What size element (kW) would be required to boil a pot of 5 litres of water in 10 minutes if the initial temperature was 16°C?***

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10. ***How much current would be drawn from a 230V supply by the water heating element in Question 9 above?***

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QUESTIONS

Answer the following questions:

12. Calculate the amount of work done if a force of 520N has to be exerted to move a body a distance of 4.3m.

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13. What power is being used if the operation (in 12) is done in (a) 5 seconds and (b) 30seconds?

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14. The work done in shifting a block of steel 2m is 510J. What force has to be exerted on the block?

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15. ***A motorcycle is push started at a constant 4.5m/s for 4seconds. If the power expended is 600W, what is the force exerted on the handle bars?***

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16. ***A force of 150N acts at 60º East to a force of 200N which is at 0º(North) . Using the parallelogram method, determine graphically the resultant value and direction of the resultant force with respect to the 200N force.***

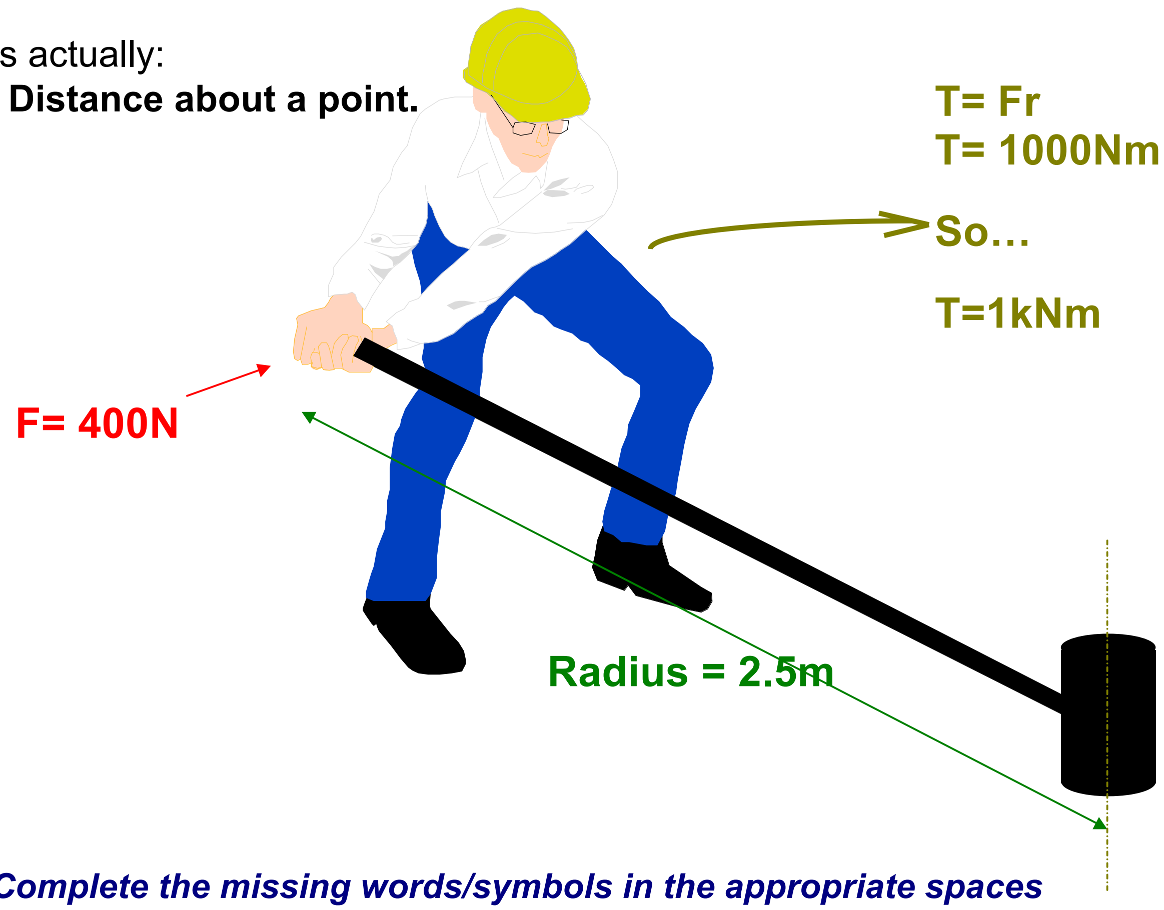
17. Two forces of 28N @ 0º(North) and 170N (Eastwards) act at right angles to each other. Calculate the resultant value of force and direction being exerted. Wrt to the 28N force.

18. Two forces 90º apart (North and Eastwards) each of 35N, act outwards at a point. Find the resultant force and angle at which it acts wrt to the north force.

19. Using the polygon method, find graphically the resultant of the following three forces, acting at 120º from each other at a single point. 180N (@0º), 400N, then 25N.

L. TORQUE

Torque is actually:
Force × Distance about a point.



11. Complete the missing words/symbols in the appropriate spaces below:

Torque is measured in:

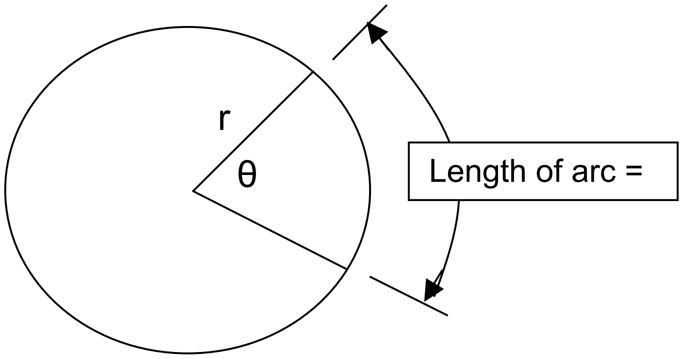
The symbol of these units is:

Note: An important thing to remember about torque is that a pulley that had a weight attached to it would have a certain amount of Torque, but would not do any Work if the weight was not heavy enough to turn the pulley - and although Work can be calculated from Newton-metres (i.e., 1Nm = 1J), this is not the case for Torque as 1Nm of Torque ≠ 1joule since there is no movement.

ANGULAR VELOCITY

When dealing with a rotating body or quantities derived from rotation, it is necessary to consider angular velocity, that is, the angle through rotation occurs in a given time. Angular velocity is expressed in radians per second (rad/s).

A radian is the angle subtended by an arc whose length is equal to the radius. In this diagram θ = 1 radian, r = the radius = length of arc.



Since the circumference C is found from C = 2 π r, it follows that there are 2 π radians in one complete revolution. i.e. 360º = 2 π radians.

20. ***A 400mm dia steering wheel requires 15N to turn it. If it were replaced with a 300mm dia wheel, what force would be required to turn it?***

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21. ***A 50kg sack hangs at rest from a rope wrapped around a 150mm dia. pulley. What is the torque about the axis of the pulley?***

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22. ***What is the angular speed of each of the three hands of a 12 hour clock?***

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23. ***If 1 joule = 1 watt × 1 second, how many joules are in a "unit" (kilowatt-hour) of electricity?***

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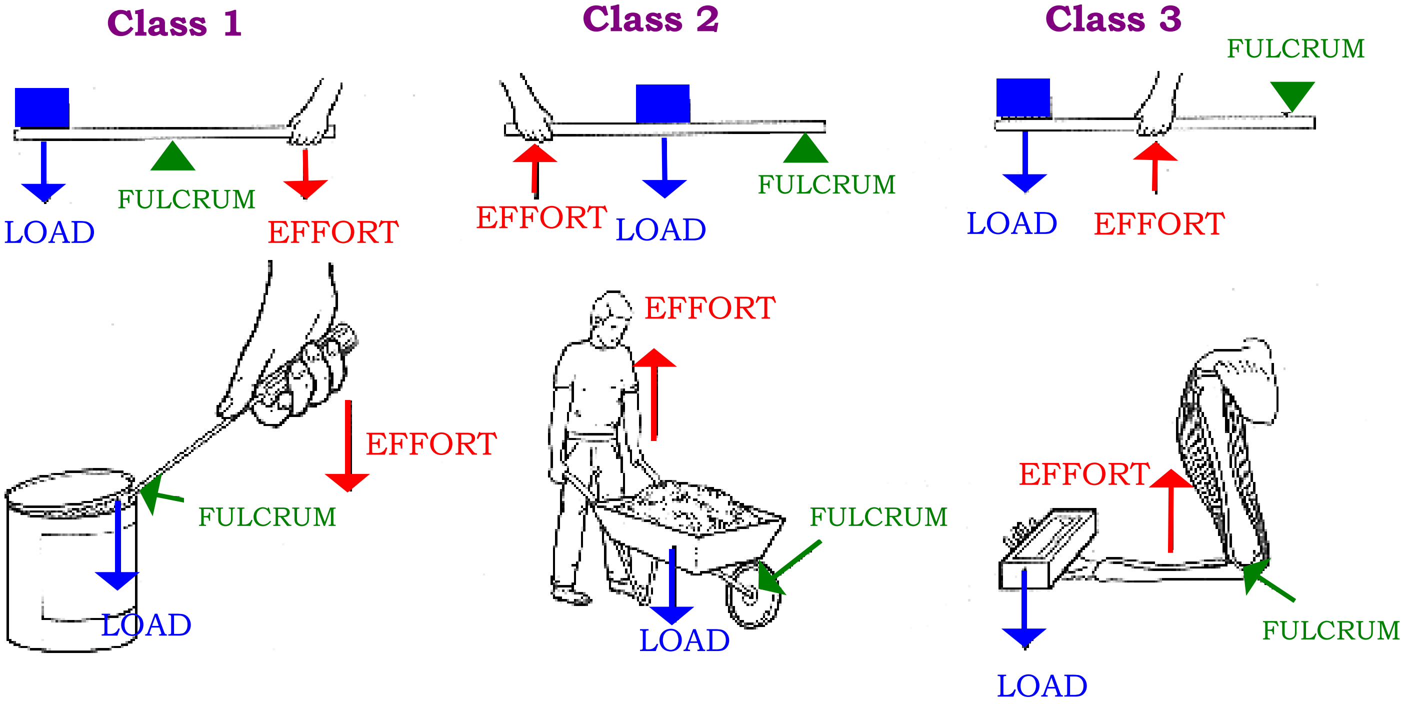
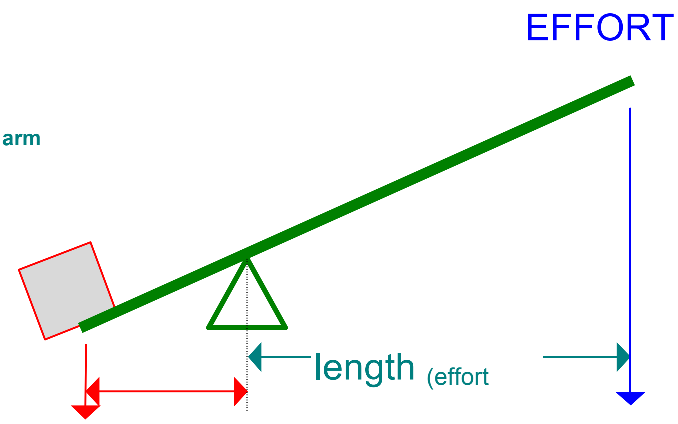
2 - Lever's & Mechanical Drives

3 SIMPLE LEVERS

The three classes of lever are illustrated below:



CLASS 1 LEVER CALCULATIONS



If the effort force × perpendicular length$_{\text{effort arm}}$

is greater than the

load force × perpendicular length$_{\text{load arm}}$

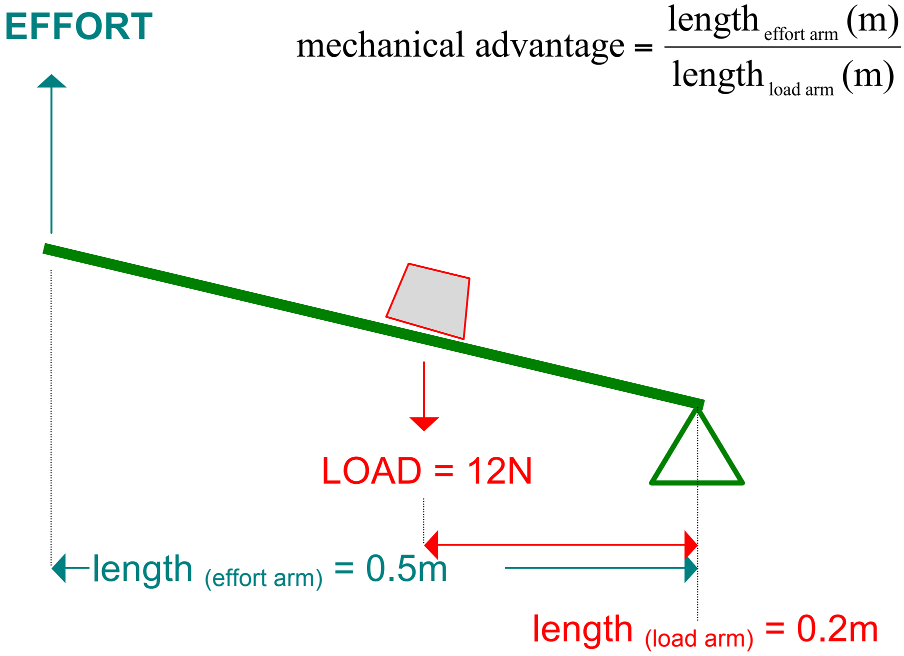
the load will be lifted.

The "mechanical advantage" of this type of lever is determined by $\frac{\text{length}_{\text{effort arm}}}{\text{length}_{\text{load arm}}}$.

CLASS 2 LEVER CALCULATIONS

The "mechanical advantage" as mentioned at the bottom of the previous page can also be used to calculate the minimum effort required to hold the load at equilibrium (i.e., any more effort will lift the load)...

$$\text{mechanical advantage} = \frac{\text{length}_{\text{effort arm}}\ (\text{m})}{\text{length}_{\text{load arm}}\ (\text{m})} = \frac{\text{load force (N)}}{\text{effort force (N)}}$$



The Force that would have to be applied to hold this Load at equilibrium (in a steady state) can be calculated from the mechanical advantage of this Class 2 Lever:

$$\text{Mechanical advantage} = \frac{\text{length}_{\text{effort arm}}\,(\text{m})}{\text{length}_{\text{load arm}}\,(\text{m})}$$

$$= \frac{0.5\text{m}}{0.2\text{m}}$$

$$\text{Mechanical advantage} = 2.5$$

$$\text{Effort} = \frac{\text{Load}}{\text{Mechanical advantage}}$$

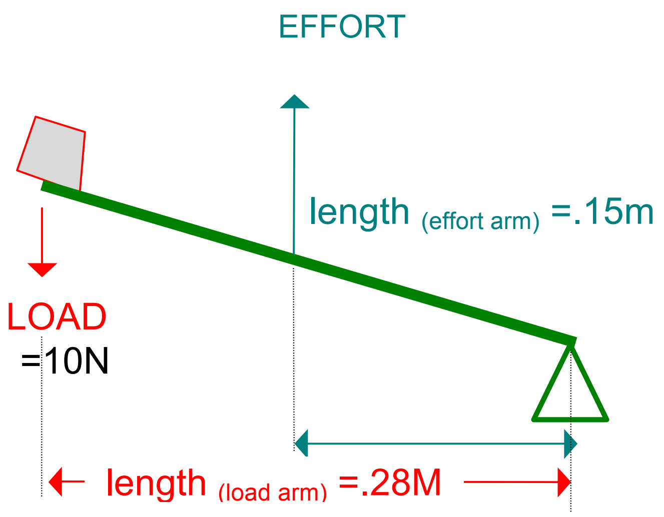
$$= \frac{12\text{N}}{2.5}$$

$$\text{Effort} = \underline{\underline{4.8\text{N}}}$$

CLASS 3 LEVER CALCULATIONS

Calculating the Mechanical Advantage is probably the easiest way to determine the minimum Force required to hold the load at equilibrium -

In both the Class 1 and Class 2 lever calculations, if we multiply the Effort Force by the Mechanical Advantage the result is the Load Force.



It is probably more practical that we divide the Load Force by the Mechanical Advantage to calculate the minimum Effort Force required to hold the Load at equilibrium:

$$\text{Effort} = \frac{\text{Load}}{\text{Mechanical Advantage}}$$

$$\begin{aligned}\text{Mechanical advantage} &= \frac{\text{length}_{\text{effort arm}}(\text{m})}{\text{length}_{\text{load arm}}(\text{m})}\\ &= \frac{0.15\text{m}}{0.28\text{m}}\\ \text{Mechanical advantage} &= 0.5357\end{aligned}$$

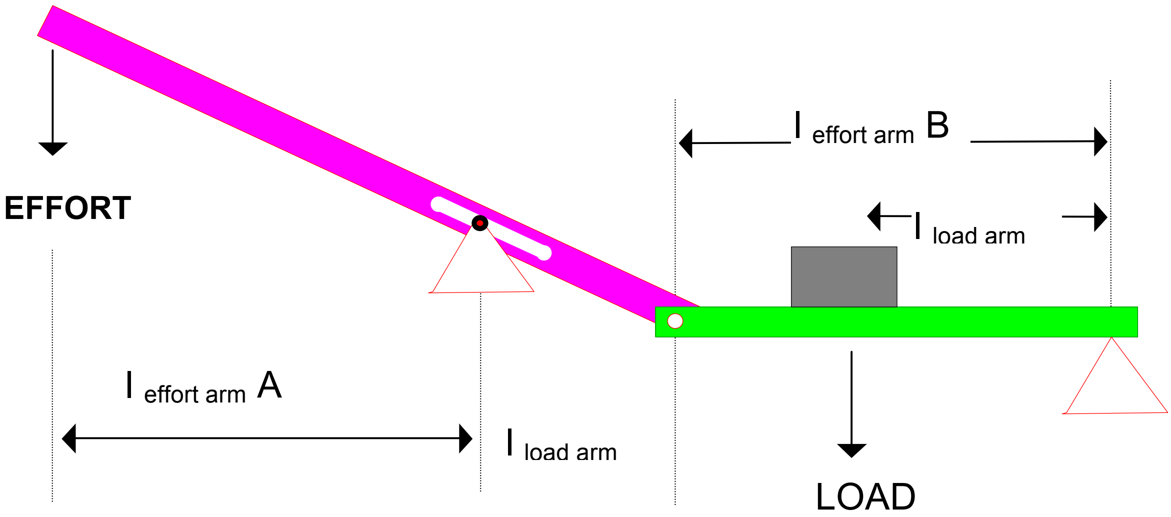
$$\begin{aligned}\text{Effort} &= \frac{\text{Load}}{\text{Mechanical advantage}}\\ &= \frac{10\text{N}}{0.5357}\\ \text{Effort} &= \underline{\underline{18.67\text{N}}}\end{aligned}$$

Obviously then, it takes more Effort than Load to raise the Load of a Class 3 Lever.

COMPOUND LEVERS

The name "compound lever" would suggest that a simple machine consisting of 2 or more levers would be a compound lever.

To solve for the Effort required it is practical (easier) then to calculate the Mechanical Advantage of each lever, multiply them together to get the Total MA and then find the Effort required.



Answer the following question:

24. How much EFFORT would be required to hold this LOAD at equilibrium if:

a) the LOAD had a 15kg mass

b)the length of $_{load\ arm}$ A is 0.2m, and, the length of $_{effort\ arm}$ A is 0.5m

c)the length of $_{load\ arm}$ B is 0.3m, and, the length of $_{effort\ arm}$ B is 0.4m

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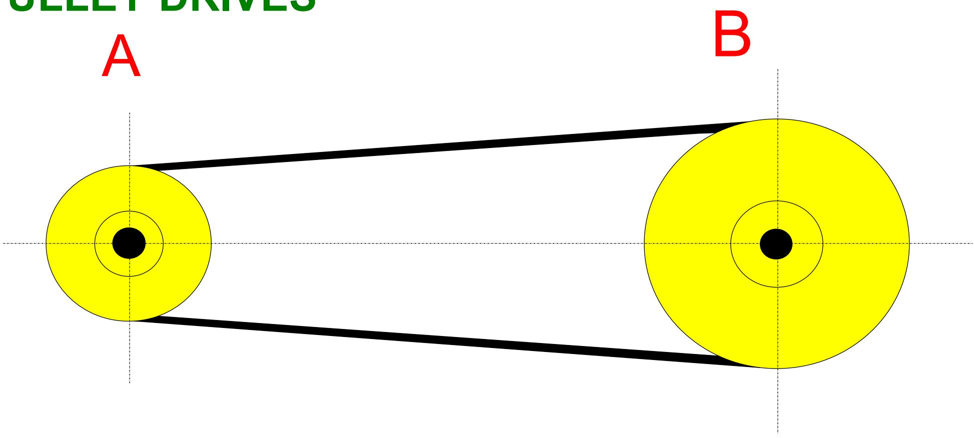
BELT-AND-PULLEY DRIVES



Belts and Pulleys are often used to mechanically couple electric motors to the mechanical loads they drive. The main use of this system is to transfer more torque to the load than is readily available from a high-speed electric motor.

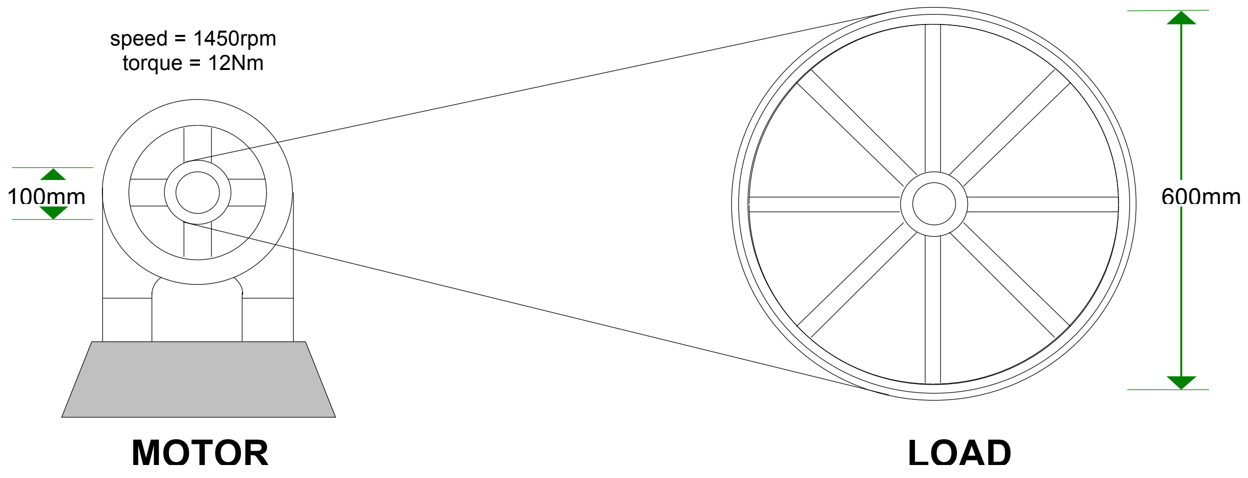
A motor with a small pulley will drive a load connected to a larger pulley at a slower speed than the motor speed - and a motor with a larger pulley than the load pulley would rotate the load faster.

The ratio of the speed of pulley "A" to the speed of pulley "B" in the diagram above can be found from the ratio of the diameters of the two pulleys:

$$\text{Ratio of speed} = \frac{\text{pulley "A" diameter}}{\text{pulley "B" diameter}}$$

If pulley "B" was twice the diameter of pulley "A", the speed of pulley "B" would be half that of pulley "A". The torque available from pulley "B" would however be twice that available from "A": note: Pulley "A" is the **Driver** and Pulley "B" is the **Driven.**

$$\text{Ratio of torque} = \frac{\text{pulley "B" diameter}}{\text{pulley "A" diameter}}$$



In the diagram above:

If an electric motor driving the small pulley rotated at 1450rpm (revolutions per minute), and the diameter of the small pulley was 100mm; the speed of the large 600mm pulley can be calculated:

$$\text{if motor speed} = 1450\text{rpm}$$

$$\text{Load speed} = \text{motor speed} \times \left(\frac{\text{motor pulley diameter}}{\text{load pulley diameter}}\right)$$

$$\text{Load speed} = 1450 \times \frac{100\text{mm}}{600\text{mm}} = 241.7\text{rpm}$$

The amount of torque transferred from the motor pulley to the load pulley is inversely proportional to the change in speed:

$$\text{if motor torque} = 12\text{Nm}$$

$$\text{Load torque} = \text{motor torque} \times \left(\frac{\text{load pulley diameter}}{\text{motor pulley diameter}}\right)$$

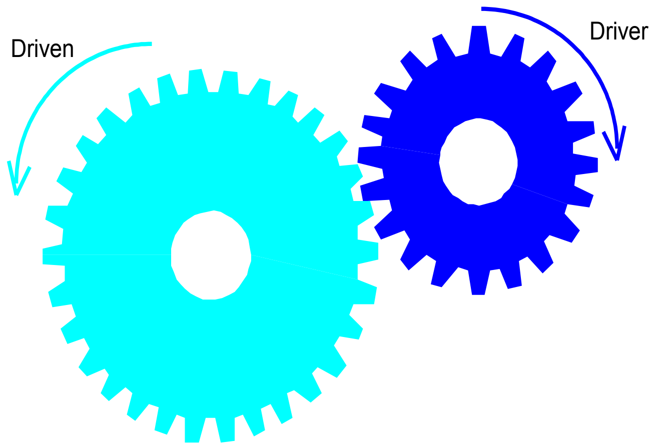
$$\text{Load torque} = 12\text{Nm} \times \frac{600\text{mm}}{100\text{mm}} = 72\text{Nm}$$

GEAR DRIVES

Just like the belt and pulley system, a gear-drive system can change the speed of the load as well as its torque. The output gear is always the same, but the driver gear to it changes. You need high torque at low speeds to initially move the car.

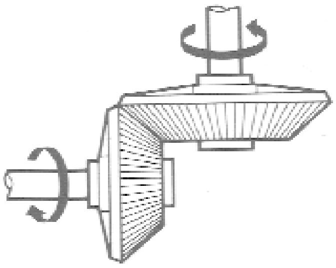
An example of this is the gear ratio of a car: 1st gear high torque: Low speed. Max speed = 185km/h ÷ 4 = 46km/h. Driving Torque = 137Nm x 4 = 548Nm.

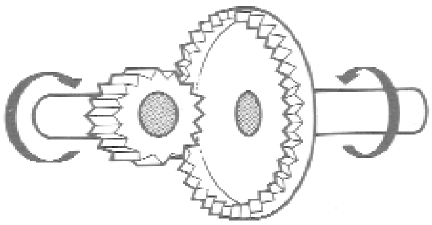
	1st gear	2nd gear	3rd gear	4th gear
Gear Ratio	4 to 1	2.5 to 1	1.5 to 1	1 to 1
Max. Speed	46km/h	74km/h	123km/h	185km/h
Driving Torque	548Nm	343Nm	206Nm	137Nm

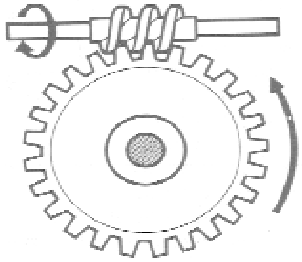


The gap between teeth on a gear drive has to be the same, otherwise the gears wouldn't be able to mesh together. The diameter of the gear would determine how many teeth could fit around that gear - so we can safely assume that a gear with 30 teeth would spin at a lower speed of a gear with 20 teeth, if those two gears were meshed. The bigger gear would have more torque, by the same ratio.

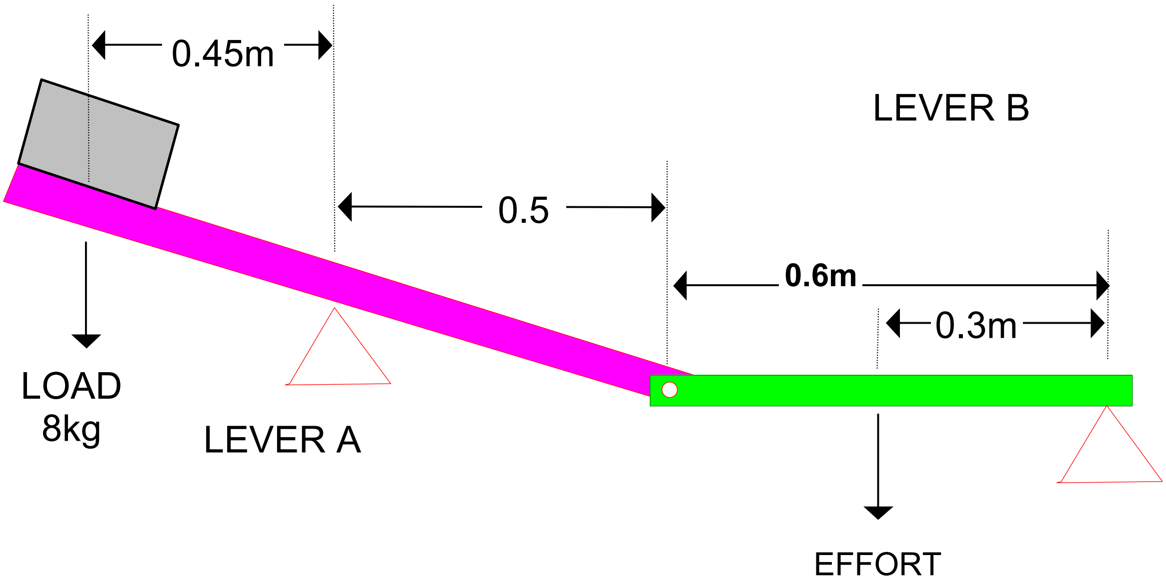
Gears have the added advantage over belts and pulleys, that they are a mechanically stronger. Belts and Gears both can change the direction in which the power is transmitted.


Bevel Gears


Crown Wheel Gear


Worm Gear

25. Calculate the Effort required to hold this "Compound Lever" at equilibrium.



26. What are the 2 classes of lever in the compound lever above?

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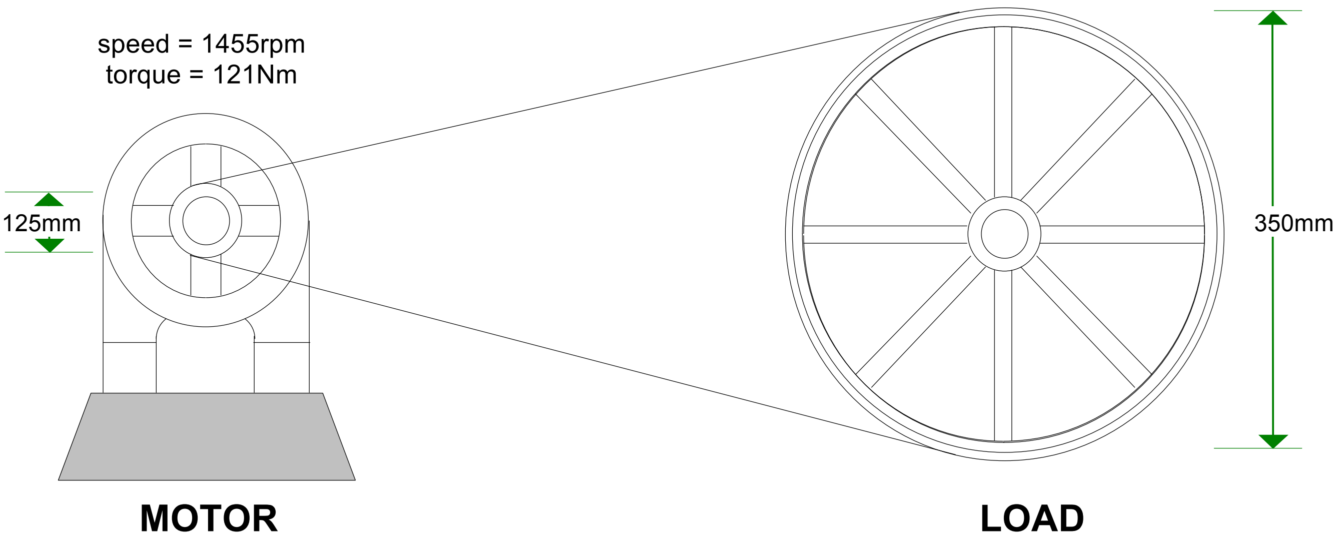
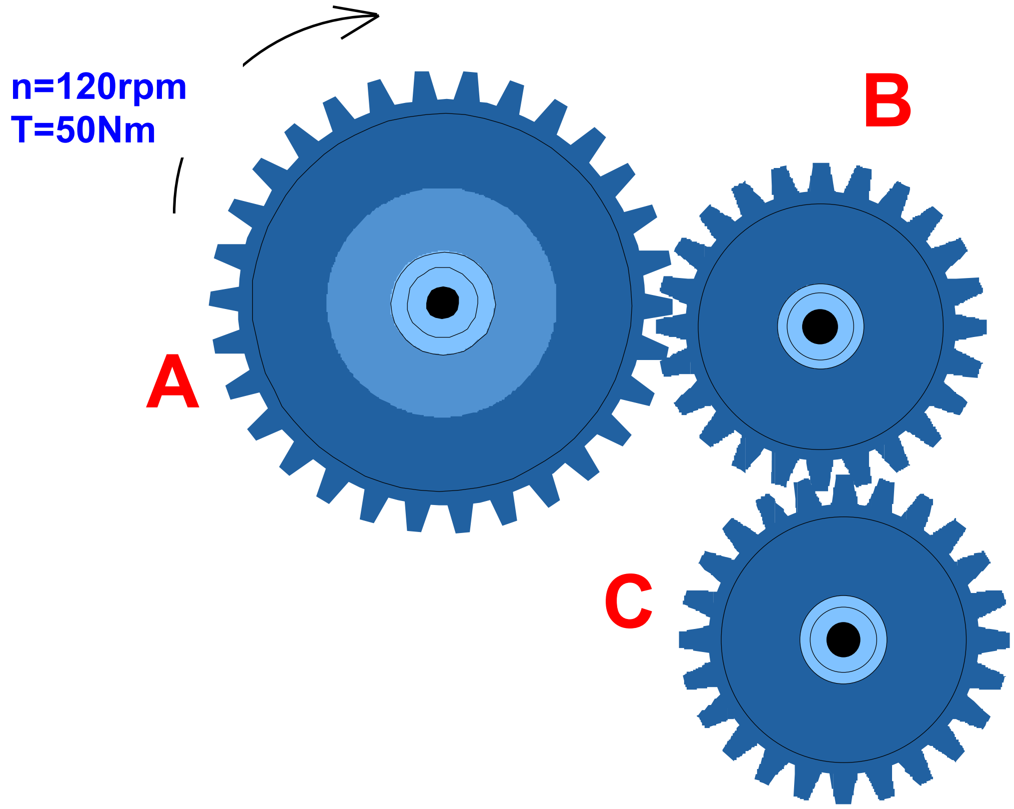
27. The "rated torque" of an electric motor is its "shaft torque". A pulley on that shaft would have also have the same torque as the speed is the same.

The rated speed and torque of a motor is 1455rpm and 121Nm, and the pulley on the shaft was 125mm in diameter. What would the rotational speed of a 350mm load pulley, and how much torque would be available at the load?



28. How much Torque would Gear "C" have, how fast would it be rotating (in revs per min), and in which direction (clockwise or anticlockwise)?



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29. Complete the following car gearbox Torque and speed ratio:

	1st gear	2nd gear	3rd gear	4th gear
Gear Ratio	4 to 1	2.5 to 1	1.5 to 1	1 to 1
Max. Speed				220km/h
Driving Torque				160Nm

EFFICIENCY CALCULATIONS

The **efficiency** of a machine is another factor that must be taken into account.

The efficiency of a small motor is poor, and larger motors 18.5kW to 315kW have efficiencies from 89 - 95%.

A typical electric storage hotwater system has an efficiency of between 80 - 85% and the efficiency of a gas storage hotwater system may be as low as 65% due to heat energy being wasted up the flue. Extra lagging on an electric storage hotwater cylinder could increase its efficiency to above 90%.

The efficiency of a belt and pulley system is similar to that of a gear-drive or chain-drive and is usually greater than 95%.

Quite simply:

$$\text{efficiency}\ (\eta) = \frac{\text{output power}}{\text{input power}}$$

If the efficiency of a small motor was 85%, and the efficiency of the belt-and-pulley system coupling that motor to the mechanical load was 98% - the efficiency of the system can be calculated as follows:

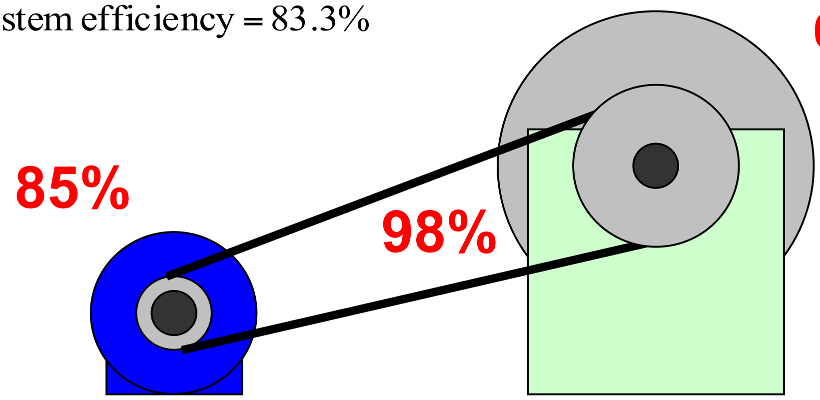
$$\text{system efficiency} = \text{motor efficiency} \times \text{belt} - \text{and} - \text{pulley efficiency}$$

$$= 85\% \times 98\%$$

$$\text{system efficiency} = 83.3\%$$

$$0.85 \times 0.98 = 0.833$$

$$0.833 \approx 83.3\%$$



Complete the following questions…

30. A hot water system would require 25MJ to heat water to a given temperature if it was a 100% efficient. How much Energy would the hot water system require if that system was only 82% efficient?

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31. The "Power Rating" on a motor is the "maximum output power". How much power would be required by a 2.2kW motor if it had an efficiency of 78%?

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32. How much torque must be supplied by a motor through a 71mm∅ pulley coupled by belt-and-pulley, if the Load pulley is 150mm∅ and the torque required by the load is 26Nm and the belt-and-pulley coupling has a 97% efficiency?

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33. How much power would be required by a 3kW electric motor (η_{motor} = 83%) if it was coupled to a mechanical load by a gearbox ($\eta_{gearbox}$ = 99%)?

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3 – Energy & Power

JOULES, WATT-SECONDS AND KILOWATT-HOURS

We have already mentioned that 1Joule of Energy = 1 watt × 1 second.

Another derived unit of electrical energy is the **kilowatt-hour** - this is normally what appears on your electricity bill as a "unit" (in 1999 a unit cost $0.135).

$$\begin{aligned}\text{Joules per kilowatt}-\text{hour} &= \frac{1\text{kilowatt}-\text{hour}}{1\text{watt}-\text{second}}\\ &= \frac{1000\text{W}\times 60\text{min}\times 60\text{sec}}{1\text{watt}\times 1\text{sec}}\\ &= \frac{3{,}600{,}000}{1}\\ \text{Joules in a kilowatt}-\text{hour} &= \underline{\underline{3.6\text{MJ}}}\end{aligned}$$

There are 3.6MJ in a kilowatt-hour.

If you divide joules by seconds you get watts - and likewise - if you divide kilowatt-hours by hours you get kilowatts.

Example:

A hotwater cylinder requires 57MJ of electrical energy to heat water to a given temperature. How long would it take the cylinder to carry out this task if the power rating of the element was 2kW?

$$\text{kWh} = \frac{57\text{MJ}}{3.6\text{MJ/kWh}} = 15.833\text{kWh}$$

$$\text{time (hours)} = \frac{15.833\text{kWh}}{2\text{kW}} = 7.917\text{h}$$

$$\text{time} = 7\text{hours} + 0.917\times 60\text{mins} = \underline{\underline{7\text{hours}, 55\text{minutes}}}$$

ENERGY AND TORQUE

1Nm of rotating torque is not equivalent to 1joule of Energy. Rotational Energy is actually in Newton-metre-radians
One complete rotation = 360° = 2π radians.

Example

How much torque would be produced by a 15kW motor that has a rated speed of 960rpm?

$$\text{Power} = \frac{2\pi \text{NT}}{60}$$

$$\text{Torque} = \frac{\text{Power} \times 60\text{sec}}{2\pi \times \text{speed(rpm)}}$$

$$= \frac{15000 \times 60}{2\pi \times 960}$$

$$= \underline{\underline{149.2\text{Nm}}}$$

How much Work can be done by this motor in 1 minute?

$$\text{Workdone} = \text{Torque} \times \text{angle turned through in radians}$$

$$= 149.2\text{Nm} \times 2\pi \times 960\text{rpm}$$

$$= 900\text{kJ}$$

The Work Done by this motor in 1 minute can also be calculated from:

$$Power \times 60\sec = 15000W \times 60\sec = \underline{\underline{900kJ}}$$

Answer the following questions…

34. ***If 1MJ of Energy is required to move a volume of coal on a conveyer, how long would the 15kW motor (previous) take if geared 80% efficiently to the conveyer?***

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35. ***A 5 kW motor rotates at 960 RPM on full load.***
(a) What torque is being exerted?
(b) If the motor has a pulley of 200 mm diameter, what force is being exerted at the circumference of thy pulley?

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36. ***What would be the efficiency of the motor in problem 35 above if the input power was 5.3 kW?***

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37. ***An electric motor running at 1200 rpm is supplying a torque of 5 N m. What power is the motor consuming from the mains at 78% efficiency?***

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38. ***A 5 kW motor has an operating efficiency of 71 %. What is the input power requirement?***

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39. ***A 300 kg weight is lifted with a block and tackle to a height of 4 m in I minute and 20 seconds.***
(a) Neglecting friction, calculate the power used.
(b) What is the efficiency if the actual power used is 24 W?

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40. A hotwater cylinder requires 57MJ of electrical energy to heat water to a given temperature. How long would it take the cylinder to carry out this task if the power rating of the element was 1.5kW?

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CONVERSION TABLE

1 inch	=	25.4 millimetres
1 foot	=	30.48 centimetres
1 yard	=	0.9144 metres
1 mile	=	1.609344 kilometres
1 nautical mile	=	1.852 kilometres
1 cubic inch	=	0.016387 litres
1 pint	=	0.5682613 litres
1 gallon	=	4.54609 litres
1 acre	=	0.4046856 hectares
1 ounce	=	28.349523 grams
1 pound	=	0.4535924 kilograms
1 stone	=	6.3502932 kilograms
1 ton	=	1.01604 metric tonnes
1 horsepower	=	0.7457 kilowatts

USEFUL FORMULAE

velocity (m/s)	$v = \frac{metres}{\sec onds}$
force (N)	$F = mass \times acceleration$
force applied to a 1kg mass by acceleration due to earths gravity	$F_{(1kg)} = 9.81N$
Work done $_{linear}$ (J)	$WD = mass \times gravity \times height$
Work done $_{heating}$ (J)	$WD = mass \times SHC \times temperature\ change\ (^{\circ}C$
Work done $_{rotational}$ (J) in 1 minute	$WD = motor\ rating(kW) \times 60\sec$
Torque (Nm rads)	$T = force \times dis\tan ce$
Mechanical Advantage	$MA = \frac{perpendicular\ length\ of\ effort\ arm}{perpendicular\ length\ of\ load\ arm}$
Effort (N)	$EFFORT = \frac{Load(N)}{Mechanical\ Advantage}$
Pulley and Gear speed	$speed_B = speed_A \times \left(\frac{\varnothing_A}{\varnothing_B}\ or\ \frac{No.teeth_A}{No.teeth_B}\right)$
Pulley and Gear torque	$torque_B = torque_A \times \left(\frac{\varnothing_B}{\varnothing_A}\ or\ \frac{No.teeth_B}{No.teeth_A}\right)$

ANSWERS

1. Metres per second, m/s
2. 200kg F = m x a so m = f/a = 1962/9.81
3. Newtons, N
4. Work = 1.211kJ (1211j) , Force = 931.95N
5. 19.42J (kinetic) 1kg = 1000gm therefore 10gm = 0.01kg
6. Work Done (Energy Applied) = 2kg × 4180 × (100°C - 10°C) = 752.4kJ
7. 752.4kJ ÷ 0.95 = 792kJ (J = 11 x 9.81 x 1.8)
8. 194.2J ÷ 186.4W = 1.042s (t = J/W and W = J/t)
9. 1.7556MJ ÷ (10m × 60s) = 2.926 kW ≈ 3 kW
10. 2.926kW ÷ 230V = 12.72A ≈ 13A
11. Newton-metres, Nm
12. 520N × 4.3m = 2.236kJ
13. 2.236kJ ÷ 5s = 447.2W, 2.236kJ ÷ 30s = 74.53W
14. 510J ÷ 2m = 255N, the answer in the book is in Joules!
15. 4.5m/s × 4s = 18m, 600W × 4s = 2.4kJ or 2400Nm, 2400Nm ÷ 18m = 133.3N
16. 304.1N ∠25.29°
17. 172.3N ∠9.353°
18. 49.5N ∠45°
19. 326.4N ∠95.71°
20. T = 15N × 0.2m = 3Nm , F2 = 3Nm ÷ 0.15m = 20N

21. $T = 50kg \times 9.81m/s^2 \times 0.15m/2 = 36.79Nm$
the book does not take acceleration due to gravity into account

22. $n_{(second\ hand)} = 1/60 \times 2\pi = 104.7\times10^{-3}$ r/s
$n_{(minute\ hand)} = 1/(60 \times 60) \times 2\pi = 1.745\times10^{-3}$ r/s
$n_{(hour\ hand)} = 1/(60 \times 60 \times 12) \times 2\pi = 145.4\times10^{-6}$ r/s

23. 3.6MJ

24. Mechanical Advantage A = 0.5m ÷ 0.2m = 2.5
Mechanical Advantage B = 0.4m ÷ 0.3m = 1.333
Total Mechanical Advantage = 2.5 × 1.333 = 3.333
LOAD = 15kg × 9.81 = 147.2N Effort = 147.2 ÷3.333 = 44.16N

25. Mechanical Advantage “A” = 0.5/0.45 = 1.111
Mechanical Advantage “B” = 0.3/0.6 = 0.5
Mechanical Advantage “A × B” = 1.111 × 0.5 = 0.5556
Force on Load = 8kg × 9.81 = 78.48N
Effort Force = 78.48N ÷ 0.5556 = 141.3N

26. Class I and Class III

27. Speed = 1455rpm × 125/350 = 520rpm
Torque = 121Nm × 350/125 = 338.8Nm

28. A - teeth = 30 n = 120rpm, T= 50Nm –CW
B - teeth = 24 n = 150rpm, T= 40Nm -ACW
C - teeth = 24 n = 150rpm, T= 40Nm - CW

29. 1st = 55km/h & 640Nm, 2nd = 88km/h & 400Nm, 3rd = 146.7km/h & 240Nm

30. 25MJ ÷ 0.82 = 30.5MJ

31. 2.2kW ÷ 0.78 = 2.821kW

32. Torque req. = (26Nm/0.97) × (71/150) = 12.69Nm

33. Power in = 3kW ÷ (0.83 × 0.99) = 3.651kW

34. T = 1MJ/15kW × 0.8 = 83s

35. $P=2\pi NT/60$ - $T=P60/2\pi N$

$T = 5000 \times 60 / 2\pi \times 960 = 49.74Nm$

$T = F \times d$ - $F = T/d$

$F = 49.74Nm/0.1m = 497.4N$

36. $5kW \div 5.3kW = 94.34\%$

37. $P = 2\pi NT/60$

$= 2\pi \times 1200rpm \times 5Nm/60$

$= 628.3W$

$628.3W \div 0.78 = 805.5W$

38. $5kW \div 0.71 = 7.042kW$

39. $WD = 300kg \times 9.81 \times 4m = 11.772kJ$

$P = WD/t = 11.772kJ/80sec = 147.5W$

615% efficient!!!

40. $57MJ / 3.6MJ = 15.833kWh$

$t(hours) = 15.833kWh/1.5kW = 10h + 0.5556 \times 60m$

$t = 10h + 33min + 0.3333 \times 60s$

$t = 10h + 33min + 20sec.$

CONGRATULATIONS !!!

FORMULAE

$$X_L = 2\pi fL \qquad MMF = S.\phi$$

$$U_m = I.N$$

$$X_L = 2\pi fL \qquad \mu = \mu_0 \mu_r \qquad P = V.I.\cos\vartheta$$

$$P = \sqrt{3}.V_{Line}.I_{Line}.\cos\vartheta$$

$$X_C = \frac{1}{2\pi fC} \qquad f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \qquad L = \frac{\mu_0 \mu_r N^2 a}{l}$$

$$\Phi = B.a \qquad SHC_{H^2O} = 4180 \qquad \varepsilon_0 = 8.854 \times 10^{-12} F/m$$

$$C = \frac{\varepsilon_0 \varepsilon_r A(n-1)}{d}$$

$$E = B.l.v.\sin\vartheta \qquad \mu_0 = 4\pi \times 10^{-7} H/m$$

$$\frac{R_1}{R_2} = \frac{1+\alpha.t_1}{1+\alpha.t_2} \qquad S = \frac{l}{\mu_0 \mu_r a} \qquad \rho_{Cu} = 1.72 \times 10^{-8} \Omega m$$

$$f = \frac{B^2 a}{2\mu_0}$$

$$S = O/H \quad C = A/H \quad T = O/A$$